EXPLORING THE PERNICIOUS ERRORS IN ALGEBRA PROBLEM-SOLVING AMONG FRESHMEN BSED-MATHEMATICS STUDENTS OF NORTHERN BUKIDNON STATE COLLEGE

[1]*Loisel B. Buelban and [2]* Rosie Tan
University of Science and Technology of Southern Philippines, Cagayan de Oro City, Philippines
*Correspondence Email: loiselbuelban@gmail.com

ABSTRACT: *This study examined the pernicious errors encountered by first-year BSEd-Mathematics students when solving algebra problems. The aim was to address their difficulties and offer strategic teaching and learning interventions for the future, when they become mathematics teachers themselves. This study employed a qualitative descriptive approach. The participants were administered a set of 6 classic word problems commonly encountered by students during their secondary education, covering topics such as percent, numeracy, age, mixture, consecutive integers, and work problems. This study employed a qualitative descriptive research design. The qualitative part of this study was utilized to conduct a content analysis to examine the students' problem-solving errors, while the descriptive component was employed to describe the frequencies and percentages characterized by those errors.*

The findings revealed that conceptual errors were more prevalent among the participants than procedural errors. The results showed that the participants exhibited a pattern of compounding errors and encountered a significant impediment in the following stages of problem-solving: defining variables, translating sentences into symbolic representations, and formulating the equation. Hence, this study underscored the importance of addressing these errors and implementing targeted future interventions to enhance problem-solving skills among prospective math educators.

Keywords: Algebra problem-solving, conceptual error, procedural error, BSEd-Mathematics, conceptual knowledge, procedural knowledge, misconceptions, Newman's Error Analysis, Polya's problem-solving framework

1. INTRODUCTION

Word problems are innately difficult to comprehend because they are often a new entity that even mathematically skilled ones greatly struggle with it [1]. Problem-solving in mathematics can be a particularly challenging course not only because it introduces more abstract representations and more complex relationships between quantities and variables [2]. The Program for International Student Assessment (PISA) has found that many students globally struggle to demonstrate proficiency in mathematics, especially when it comes to applying their knowledge to solve real-world problems [3]. Students often struggle to convert word problems into mathematical expressions. This includes difficulties in understanding the language used in the problems and translating it into mathematical terms [4]. This suggests that there is a systemic issue in how mathematical concepts are taught and understood at various educational levels. Moreover, research indicates that students face significant challenges in solving word problems, particularly in the context of algebra. A study conducted on 51 Indonesian students revealed that formulating mathematical models is a primary difficulty, with many students making errors in creating equations, schemas, or diagrams necessary for problem-solving. This also aligns with findings from the Trends in International Mathematics and Science Study (TIMSS), where only 8% of Indonesian students could successfully solve a specific word problem, compared to an international average of 18% [3]. Studies and data have shown that difficulties in solving algebraic word problems persist as a challenge not just for students in the Philippines, but rather represents an academic global challenge Consequently, for freshmen students pursuing a Bachelor of

Science in Mathematics education degree, a mastery even in the fundamentals of algebra is very crucial [5]. As future mathematics educators, these students must be adept at transforming real-world situations into algebraic equations and then solving them, a skill that is central to working through algebra word problems [6]. Mastering this skill not only lays a strong foundation for advanced mathematical concepts but also equips aspiring teachers with the necessary tools to effectively teach and support their future students [7]. Another reason why mastery in algebra word problem solving is essential for BSED Mathematics freshmen students is the strong foundation it provides for their future academic and professional success. This will help them prevent the repeating cycle of problems that their future students may face. By mastering fundamental algebra concepts, prospective mathematics education majors will be better equipped to break down complex algebraic ideas and provide clear explanations to their students. This will ensure that their future students do not experience the same struggles and difficulties when learning to solve algebra word problems, leading to a deeper understanding and better long-term outcomes [8]. Hence, a college student pursuing a major in mathematics education is expected to gain at least a mastery on the essential pre-requisite skills of algebra because they will be performing a crucial role in training the minds of their learners in the future when they themselves become a math teacher. A teacher's knowledge is an important of component of teaching [9]. The teacher's expertise and knowledge in the subject matter is a crucial element of instruction and has a significant impact on how well the students learn. Students benefit from the teacher's content and pedagogical knowledge in mathematics as well as in their pedagogical expertise. Determining the common errors and specific difficulties that BSED-Mathematics freshmen encounter when solving algebraic word problems is essential. This knowledge can then be used to develop more targeted and effective instructional strategies, teaching methods, and interventions to improve students' problem-solving skills and conceptual understanding in this critical area of mathematics. Ultimately, this places the researcher in a position to provide insights for students and teachers alike to explore possible efficient teaching and learning methods and future interventions to address the challenges in solving algebra word problems.

1.1 Theoretical Framework

This framework draws upon two prominent theories of Polya's Problem-Solving Theory and Newman's Error Analysis, paralleled together to provide a comprehensive lens for understanding and analyzing student errors. The following are the different stages involved in Polya's and Newman's when solving word problems in algebra.

Reading Errors vs. Understanding the Problem.

In Polya's framework, "understanding the problem" is crucial. It involves identifying key information like identifying the given, defining the unknowns, and understanding the conditions. In this stage. students may completely understand the problem. However, more often than not, students lacked the necessary conceptual knowledge to arrive at the correct answer. Reading errors occur when a student fails to understand the given, incorrectly defines the variables, and fails to make logical connections and relationships between the given and variables. Moreover, the connection between understanding and reading error is that understanding the problem is the desired outcome of effective reading and analysis.

Transformation Errors vs. Devising a plan.

Polya's "devising a plan" involves the process of translating sentences into mathematical symbols, and making connections or relationships between the given and the unknowns [10]. In addition, the student is also able to devise a plan by selecting the right formula to correctly solve the problem [11]. On the other hand, transformation error occurs when a student understands the problem's requirements but struggles to translate that understanding into a workable mathematical equation. In short, transformation errors directly impede a student's ability to "devise a plan" effectively. A student might understand the problem's objective but stumble when trying to translate that understanding into a concrete plan of action.

Process Skills Errors vs. Carrying out the plan.

Polya's "carrying out the plan" involves executing the chosen strategy. This means applying the formula, algorithm, or steps identified in the devising or planning stage. Applying the appropriate formula and processing it are both essential steps for arriving at the correct solution. In other words, this stage involves solving the formula or equation that was selected in the preceding step. Performing calculations accurately, requiring attention to detail and fluency with mathematical operations. In contrast, Newman's "process skills errors" occur when a student makes errors in applying mathematical procedures or incorrectly manipulating equations, losing track of mathematical steps or procedures or misapplying order of operations and the like. Process skills errors directly undermine a student's ability to "carry out the plan" effectively.Even with a sound plan, errors in execution will lead to an incorrect solution.

Encoding Errors v.s. Looking back.

Polya's "looking back" encourages reflection and sense-making after a solution is reached which involves checking the answer if the solution makes sense, verifying the solution for any errors, considering other alternative methods if the problem is solved differently, thinking if the solution can be applied to similar problems. Meaning the "looking back stage" of Polya, requires verifying the solution before considering it final, ensuring it is free from any kind of errors. While encoding errors may occur when a student arrives at a correct mathematical solution but struggles to translate that solution back into the context of the original problem such as providing an answer with incorrect or missing units, failing to answer the specific question asked, not recognizing the reasonableness of the answer or overlooking unrealistic solutions that don't align with the problem's context. Encoding errors directly hinder a student's ability to effectively "look back" and demonstrate true understanding. "Looking Back" is about more than just checking for calculation errors; it's about ensuring the solution makes sense within the broader problem situation. Below is a table that shows the parallel comparison and contrast of the stages of Polya's problem-solving theory.

Parrallel comparison of stages in problem-solving Polya's Problem-solving Theory vs. Newman's Error of Analysis

Newman (1977)	*Polya (1981)*
i. Reading Error	
ii. Decoding Error	*i. Understanding the problem*
ii. Transforming Error	*ii. Devising a plan*
iv. Processing Error	*iii. Carrying out the plan*
iv. Encoding Error	*iv. Looking back*

In this study, each word problem was divided into 4 stages based on Polya's problem solving framework: understanding the problem, defining the variable, translating sentences into symbols, and solving the equation. Then the errors in these worked problems will then be analyzed based on these four stages of Polyas**.** Moreover, each stage of Polya's problem-solving framework has a corresponding equivalent stage in Newman's Error Analysis, as shown in a parallel comparison table. Newman's error analysis is then used to identify the corresponding errors that students have made in relation to each stage of Polya's problem-solving framework. The ultimate goal of comparing Newman's error analysis and Polya's problem-solving framework in parallel is to have a more comprehensive understanding of the types of errors students make at each stage of the problem-solving process.

1.2 Research Questions

1. What stages in problem-solving do students mostly commit errors based in Polya's and Newman's frameworks?
2. What specific are the pernicious errors do students make in each type of word problem?

2. METHODOLOGY

2.1 Research Design

This study utilized a qualitative descriptive approach, specifically content analysis, to deeply explore the nature and characteristics of errors made by students when solving algebra word problems. On the other hand, descriptive statistics were employed to present and describe the frequency and distribution of errors across the different stages of the problem- solving process, as well as the various types of word problems. The researchers employed Newman's Error Analysis as the basis for analyzing and categorizing the errors that students exhibited across the various stages of the problem-solving process.

3.3 Sampling Technique

This study employed a total population sampling technique, a form of purposive sampling method that examines the entire population. This approach is particularly effective when the population is small and manageable, allowing for comprehensive data collection. The population for this study comprised of 41 freshmen Bachelor of Secondary Education major in Mathematics students from the Teacher Education Program at Northern Bukidnon State College.

3.4 Research Instrument

The research instrument consisted of six algebra word problems designed to assess students' problem-solving skills. The problem's level of difficulty varied from easy to average, as they were tailored to the mathematical abilities of the participants, who were of low to average mathematical abilities. The selected word problems cover a variety of common algebraic topics, including: age, mixture, percentage, numeracy, consecutive integers and work problems. These specific word problem types were chosen due to their prevalence in algebra curricula and their documented difficulty for students at various educational levels [12]. Moreover, word problems frequently appeared in college entrance exams and standardized tests, such as the SAT and ACT, where they assessed students' mathematical reasoning and comprehension. For instance, the SAT included a section dedicated to problem-solving and data analysis, which often featured word problems that required students to interpret information and apply mathematical concepts which is the reason why these sets of problems were chosen.

3.5 Instrument Validation

Two expert colleagues with expertise in algebra and problem-solving evaluated the instruments. The six word problems were assessed and found to have difficulty levels that ranged from easy to moderate. The researcher-developed rubrics used a three- point scale based on four criteria. The first criterion, linguistic complexity, considered whether the word problems used simple sentences and common vocabulary for easy readability and comprehension. The second criterion, mathematical demand, assessed whether the problems required simple or multiple-step operations. Third, contextual factors examined whether the problems had familiar and relatable contexts for the students in their daily lives. Finally, problem-solving strategies considered whether the problems called for a straightforward application of a known formula or required a non-routine problem-solving approach. In addition, the word problems were assessed for readability and understandability using the Flesch-Kincaid readability calculator [13]. This ensured the vocabulary, sentence structure, and overall language complexity were suitable to facilitate understanding [14]. Based on the Flesch-Kincaid readability analysis, 6 word problems ranged from very easy to easy, indicating they were all readily understood.

3.6 Data Collection

The data collection process for this study involved administering six algebra word problems to 41 freshmen students who majored in BSEd-Mathematics. The students were required to approach these problems using a four-stage problem-solving framework: understanding the problem, defining the variable, formulating the equation, and solving the equation. First, students carefully read and comprehended each word problem, identifying key information such as the given and the unknowns, which is a crucial first step to successfully perform the subsequent steps. Secondly, students were asked to define the variable or unknown. This means that participants were able to identify the meaningful connections between the given and unknowns, and represent an unknown quantity in the problem correctly, translating sentences into mathematical symbols. Thirdly, students were asked to formulate an equation based on their defined variables and the relationships described in the word problems. Finally, participants solved the equation they had formulated to arrive at a correct and meaningful answer. After all participants had completed answering the six problems, their responses were thoroughly analyzed to identify errors at each stage in Newman's Error Analysis. The researcher identified the common errors across all 6 word problems per stage of Newman's Error Analysis. These errors were categorized and recorded to facilitate a comprehensive analysis of student performance and common areas of difficulty. By systematically documenting errors at different stages for each problem type, the study aims to provide insights that can inform instructional strategies for improvement. Additionally, the distribution of errors across the six word problems was examined to reveal what types of problems posed greater challenges for the students.

3.4 Data Analysis

The participants were instructed to answer all 6 word problems by writing their responses based on the 4-stage Polya's problem-solving framework namely: understanding the problem, devising a plan, carrying out the plan, and looking back. This qualitative-descriptive study employed content analysis to identify and examine errors. It started by identifying and analyzing the harmful errors made by the students for each type of word problem. These errors were classified using the 5 stages of Newman's Error Analysis: reading error, decoding error, transforming error, processing error, and encoding error. Every unique error committed by the 41 participants in each problem at different stages of Polya's problem-solving framework was accounted for, recorded, and tabulated. There were instances where a certain error was accounted for more than once because it reflected one or more mathematical errors. In the process of identifying the diverse errors present in the participants' solutions, the researcher organized the errors based on their shared common attributes. The tabulation of those errors will then provide crucial insights, including: the unique errors made by the participants at each stage of Newman's Error Analysis, the stage of Newman's Error Analysis with the highest number of committed errors, and the type of problem-solving that accumulated the most number of errors.

4. RESULTS AND DISCUSSION

4.1 Specific Errors at Different Stages Using Newman's Analysis

Stages of Errors	Word Problem Type	Specific Errors Committed for Each Problem in Different Stages	Number of Errors Committed for Each Kind	Total Number of Errors For Each Problem
Reading Error: *(Failed to understand what is asked in the problem)*	**Percent Problem**	1. Misunderstanding of the problem	**2**	**2**
	Numeracy Problem	No error	**0**	**0**
	Consecutive Integers	No error	**0**	**0**
	Age Problem	1. Grammatical errors	**3**	**8**
		2. Did not specify that what is asked is the "current age"	**2**	
		3. Misinterpretation of what is truly asked	**3**	
			1	
	Work Problem	1. No answer	**4**	**12**
		2. Lack of specific information about what is being asked	**7**	
		2. Grammatical error	**1**	
	Mixture Problem	1. No answer	**8**	**16**
		2. Misunderstanding of what is being asked	**8**	
TOTAL				***38***
Decoding Error: *(Failed to define the variables correctly.)*	**Percent Problem**	1. No answer	**1**	**4**
		2. Assigning a fix value to a variable	**2**	
		3. Ambiguous definition of variable	**1**	
	Numeracy Problem	1. No answer	**2**	**4**
		2. Ambiguous definition of variable	**1**	
		3. Assigning a fix value to a variable	**1**	
	Consecutive Integers	1. No answer	**6**	**32**
		2. Incomplete definition of variable	**25**	
		3. Assigning a fix value to the variable	**1**	
	Age Problem	1. No answer	**4**	**26**
		2. Incomplete definition of variable	**22**	
	Work Problem	1. No answer	**6**	**22**
		2. Ambiguous definition of variable	**4**	
		3. Incomplete definition of variable	**12**	
	Mixture Problem	1. No answer	**6**	**27**
		2. Ambiguous definition of variable	**8**	
		3. Incorrect variable assignment	**13**	
TOTAL				***115***
Transforming Error: *(Failed to correctly translate sentences into symbols or equation)*	**Percentage Problem**	1. Failing to add mark-up to cost price	**18**	**24**
		2. Misinterpretation of the percentage in an Equation	**5**	
		3. Incorrect value for mark-up	**1**	
	Numeracy Problem	1. No answer	**8**	**35**
		2. Incorrect Translation of "two less than a number"	**14**	
		3. Incorrect translation of " subtracted from"	**13**	
		4. Incorrect translation of " a number"	**16**	
	Consecutive Integers	1. No answer	**8**	**25**
		2. Misinterpretation of the keyword "sum of consecutive integers"	**11**	
		3. Lack of Variable Representation	**6**	
	Age Problem	1. No answer	**10**	**40**
		2. Misinterpretation of the keyword "5 years ago"	**11**	
		3. Misinterpretation of the keyword "12 years older"	**14**	
		4. Missing variable relationship	**5**	
	Work Problem	1. No Answer	**7**	**47**
		2. Incorrect combined work rate	**14**	
		3. Misinterpretation of Work-Rate relationships	**15**	
		4. Misinterpretation of individual work rate	**5**	
		5. Missing variable relationship	**6**	
	Mixture Problem	1. No answer	**6**	**19**
		2. Incorrect and incomplete representation of the quantity of bills	**11**	
		3. Missing variable relationship	**2**	
TOTAL				***190***
Processing Error: *(Failed to solve the equation correctly)*	**Percent Problem**	1. No answer	**1**	**7**
		2. Ambiguous/faulty equation	**6**	
	Numeracy Problem	1. No Answer	**3**	**19**
		2. Ambiguous/faulty equation	**16**	
	Consecutive Integers	1. No Answer	**8**	**31**
		2. Ambiguous/faulty equation	**16**	
		3. Incomplete answer	**4**	
		4. Negative signs	**3**	
	Age Problem	1.No Answer	**5**	**27**
		2. Ambiguous/faulty equation	**17**	
		3.Incomplete answer	**5**	
	Work Problem	1.No answer	**23**	**33**
		2.Ambiguous/faulty equation	**10**	
	Mixture Problem	1.No answer	**7**	**36**
		2.Ambiguous/faulty equation	**10**	
		3.Incomplete answers	**19**	
Encoding Error: (Failed to verify the final answer for correct units, for missing answers and more)	**Percent problem**	1. Missing proper units in the final answer	**41**	**82**
		2. Did not properly label the final answers	**41**	
	Numeracy Problem			
	Age Problem			
	Mixture Problem			
	Consecutive Integers			
	Work Problem			

4.2 Summary of the Distribution of Errors in Each Word Problem

		Reading	*Decoding*	*Transforming*	*Processing*	*Total*
1.	Percent Problem	2	4	24	7	***37***
2.	Numeracy Problem	0	4	35	19	***58***
3.	Consecutive Integers	0	32	25	31	***88***
4.	Age Problem	8	26	40	27	***101***
5.	Work Problem	12	22	47	33	***114***
6.	Mixture Problem	16	27	19	36	***98***
	Total	***38***	***115***	***190***	***153***	***496***

4.2.1 Reading Error

The reading error stage has been shown to have the fewest errors committed compared to the other stages evaluated in Newman's Error Analysis across all types of word problems. In this stage, most of the students understood the problem superficially but made trivial grammatical errors when expressing their thoughts on what was being asked in the problem. For the percent problem, some students struggled with the term "mark-up". However, they clearly understood what was asked in the problem which was to determine the selling price of the dining table. In the numeracy and consecutive integers problem, the students demonstrated a clear understanding of the problem allowing them to correctly understand what was being asked. In the age problem, two students struggled to understand what was being asked in the problem. Lastly, in the mixture problem, 8 students showed no answer and an additional 8 students demonstrated a lack of comprehension regarding the specific inquiry of the problem. Age problems often require reasoning about complex relationships between quantities, work problems involve challenging calculations, and mixture problems demand a deep understanding of ratios and proportions. These types of problems require strong math skills and conceptual knowledge that many students struggle to develop, leading to persistent challenges in mastering these topics. Reading errors can be due to various reasons. These include unfamiliar vocabulary that can hinder comprehension and complex sentence structures that make it difficult to understand lengthy or intricate sentences. Additionally, many students faced challenges in visualizing the problems, which prevented them from forming mental representations of the problems. They may also misinterpret the relationships between quantities, leading to errors in understanding how different elements interact within the problem context [15].

4.2.2 Decoding Error

This stage ranked second for having the fewest errors committed by participants during the problem-solving. This implied that participants defined the variables correctly for some easy problems, like percent and numeracy questions. However, this was also the stage where participants exhibited escalating errors, indicating difficulties in defining variables for problems involving consecutive integers, age, work, and mixture. Defining variables correctly is crucial for word problem-solving. When students misinterpret the problem or define variables incorrectly, it leads to flawed equations. Even if students apply math operations correctly, the results are still incorrect because the underlying relationships are flawed. Correctly defining variables allows students to clarify relationships between quantities, essential for accurate expressions and equations [16]. Specific variable meanings reduce ambiguity and confusion. Correctly defined variables enable effective communication of mathematical ideas, helping students articulate reasoning and solutions clearly [17].

Percent Problem & Numeracy Problem

For percent and numeracy problems, similar mistakes have been observed. The first error was a "no answer," which implied something that may have stemmed from a lack of understanding or comprehension of the problem. Research showed that when students struggled with understanding or comprehending the problem and became overwhelmed, they chose not to answer or left the question unanswered [18]. The second error involved assigning a fixed value, which implied that the student misunderstood the nature of the variables in the context of the problem. Variables often represent unknown quantities that can change based on the problem. When students assign fixed values to variables, which typically represent unknown quantities, it restricts their capacity to effectively explore relationships and solve for the unknowns [19]. This error highlights a gap in understanding how variables function within mathematical expressions. The third one is ambiguously defining a variable which indicated a lack of clarity about the variable's representation.

Consecutive Integers Problem

Many participants encountered difficulties in the consecutive integers problem by incompletely defining the variables. Rather than defining each integer separately. Instead, they defined the variables collectively as "Let: x- be the integers". Additionally, some students provided no answer and some have lacked variable representation. Students might try to simplify the problem by using a single variable, viewing it as a shortcut. They might think "x" can just "hold" all the consecutive values without realizing the need for separate expressions. If the concept of defining variables for consecutive integers is not explicitly taught and reinforced through practice, students may instead resort to more familiar yet erroneous approaches.

Age Problem

Two types of errors were observed for age problems: "no answers" and "incomplete definition of variables". A "no answer" is still considered an error, as it can indicate a lack of understanding the problem. Students may have been unsure how to solve it, so they skipped solving the problem entirely. Secondly, many of the students struggled with incompletely defining variables. A significant number failed to define the ages of each person separately and specifically, instead providing broad definitions such as "Let x: be the ages" or "Let: x -be the age of Carmen". Students might view a variable like "x" as a container that can hold multiple values simultaneously. They might think, "Let 'x' be the ages of the three people," not grasping that each person needs their own distinct representation. They also often neglected to specify that the age is referred to as the "present age". Students may not fully grasp the context of the problem. If the problem does not explicitly state "present age," they might assume that all ages mentioned are current without realizing the importance of distinguishing between present and past or future ages.

Mixture Problem

Similar to work and mixture problems, the most common errors were incomplete definitions of variables. They struggled to define the variable correctly due to difficulties in comprehending the mathematical terminologies used and the abstract nature of variables, which can lead to confusion and incorrect representations. Studies have shown that a significant percentage of students make decoding and transformation errors when translating word problems into algebraic expressions, indicating a lack of clarity in understanding how to define and use variables effectively [20]. In this stage, the participants have been observed to have strongly exhibited errors related to conceptual knowledge. Many had difficulty correctly defining the variables, with some defining them completely wrong and others only partially correct due to a lack of information about the variables. According to the study of [21], students struggle to understand the logic behind algebraic methods used to solve word problems. Their prior experience with arithmetic problem-solving leads them to revert to arithmetic thinking, hindering their grasp of the conceptual foundations of algebraic problem-solving. This affects their interpretation of unknowns, use of variables, and understanding of equations, as they are unable to fully transition from arithmetic to the more abstract thinking required for algebra.

4.2.3 Transforming Error

This stage exhibited the highest frequency of errors committed by the students. The conceptual knowledge required for each word problem in this stage had grown increasingly advanced and complex, resulting in a higher likelihood of mistakes. Additionally, this stage can be directly affected by any mistakes or errors committed by the participants in the previous stage. It is at this stage that students struggle to choose and set up appropriate mathematical operations or formula to arrive at a correct answer. They were unable to identify correct formulas or made errors translating sentences into algebraic expressions or sentences. Research indicated that students exhibited significant conceptual errors when translating sentences into symbolic representations in word problems. This is likely due to their poor foundational

knowledge of basic algebra concepts [22]. Several studies supported the claim that students' weak foundational knowledge in algebra hindered their ability to translate word problems into mathematical symbols, which in turn impacted their performance in advanced mathematics courses [23].In essence, a lack of strong conceptual understanding in foundational algebra adversely affected students' ability to solve problems in more advanced mathematics. Another study suggested that mathematical vocabulary played a crucial role in cognitive reasoning and could be a factor in student's ability to translate word problems into symbolic representations [24].

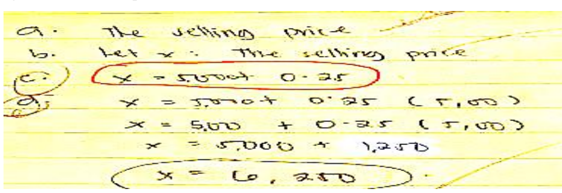
Percent Problems

Percentage problems, while fundamental to mathematics, often pose significant challenges for students. Studies have shown that several students have struggled dealing with percent due to its abstract nature. Percentages show a part-to-whole connection, which can be hard for some learners to grasp. Visualizing 25% as a portion of a whole, instead of just a number, needs deeper understanding.

a. Failing to add mark-up price to cost price

Figure 1

Failing to Add Mark-up to Cost Price

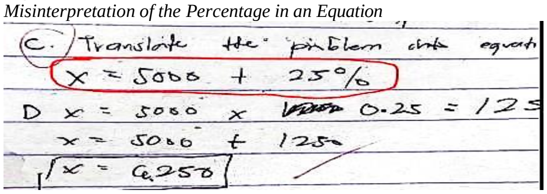


The most common errors observed in percent problems were failing to add mark-up in the cost price, which was estimated to occur in 44% of the participants of this study. This indicated that students struggled to correctly incorporate the additional costs into their calculations. The challenges students faced included difficulty conceptualizing the implications of a 20% markup on the original cost, as well as frequently confusing the cost price with the selling price, which hindered their ability to accurately apply the markup [25]. Students may have lacked a clear understanding of the interrelationships between cost price, selling price, and mark-up.

b. Misinterpretation of the percentage in an Equation

Figure 2

Misinterpretation of the Percentage in an Equation



Another mistake, is the misapplication of the % symbol inside an algebraic equation. Students tend to commonly substitute a percentage value (e.g., 25%) into an equation without converting it to its decimal equivalent (0.25). Research has indicated that many students struggled with transforming verbal statements into mathematical symbols, leading to significant errors in their calculations, particularly when interpreting and applying percentages within equations [26]

c. Incorrect value for mark-up

This error is more of a procedural error where the student mistakenly wrote the wrong value of the original cost

Numeracy Problem

a. Incorrect translation of the keyword " a number"

It was observed that 39% of the participants struggled to translate the keyword "a number" in forming a correct equation. The abstract and conceptual nature of the term "a number" may have made it challenging for students to visualize the problem concretely, leading to difficulties in translating the keyword. Also, the shift from verbal to symbolic representation can be challenging for those still developing their algebraic thinking. Students' weak reading comprehension skills and lack of experience with word problems may contribute to this issue [27].

b. Incorrect translation of the keyword " 2 less than a number"

Another common issue was students' difficulty interpreting "2 less than a number". Most had incorrectly written "2-x" instead of the accurate "x-2". This suggested learners had relied on literal, word-for-word translation rather than grasping the underlying concept. Additionally, many had misinterpreted "less than" as a cue for subtraction, writing "2 - x" instead of the correct "x - 2". This revealed a lack of understanding about the relationship between the unknown and the given difference. The abstract nature of "a number" appeared to have hindered students' ability to fully comprehend the meaning and properly formulate the algebraic expression.

c. Incorrect translation of the keyword " twelve is subtracted from five times a number"

Many participants frequently misinterpreted the mathematical expression "subtracted from" as "12-5x" rather than as "5x-12". This misconception may have stemmed from a bias in the perception of subtraction as a left-to-right process, leading to the assumption that the first number mentioned was to be written first. Furthermore, students often tended to translate sentences into mathematical symbols without fully comprehending the importance of understanding the uniqueness of mathematical language [27].

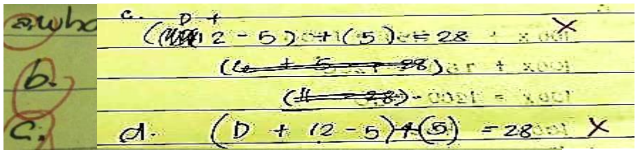
Consecutive Integers Problem

a. Misinterpretation of the keyword "sum of consecutive integers"

Some of the most prevalent errors students made in the consecutive integers problem involved a misunderstanding of the phrase "sum of consecutive integers," accounting for 26% of the participants who committed this mistake. They failed to grasp what consecutive integers were or how to correctly formulate an equation. Research showed that students often struggled to understand the concept of consecutive integers and had difficulty translating the problem statement into a mathematical expression. Many students were unsure of how to represent the sum of consecutive integers using an equation, leading to incorrect solutions [28].

Figure 3

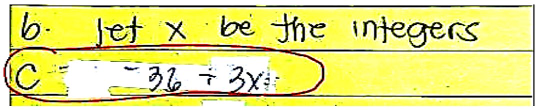
Misrepresentation of the Key Word "Sum of Consecutive Integers" as Cubed



The student's equation, reveals a critical misunderstanding of how to represent the "sum of consecutive integers" algebraically. The student seems to be associating the phrase "three consecutive integers" with the concept of cubing which is a serious conceptual error.

Figure 4

Confusing Average with Consecutive Sum

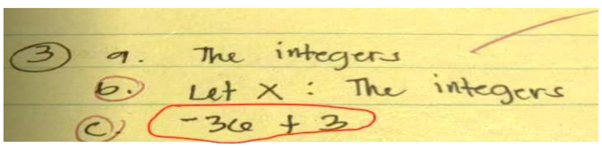


The student demonstrated a conceptual error by confusing finding an average with solving for unknown values. The student seems to be approaching the problem as if they need to find the average of three numbers.

b. Lack of variable representation

Figure 5

Lack of Variable Representation



Another error is lack of variable representation. This is another conceptual error where the student did not use variables to represent the unknown integers, making it impossible to set up an equation to solve for the unknown value, x. Students often struggled in grasping the concept of a variable as a placeholder that can represent different values. Research indicated that many learners fail to recognize that a variable can serve multiple roles, such as an unknown quantity, a generalized number, or a varying quantity, which complicates their ability to apply variables correctly in equations. This misunderstanding leads to incomplete or incorrect formulations when attempting to solve problems.

Age Problem

One of the most common errors in age problem in the transforming stage has something to do with correctly interpreting and translating keywords such as "5 years ago", "12 years older". These keywords are crucial because they signify mathematical relationships that need to be represented accurately in equations. Approximately one-quarter of the participants opted not to answer the problem due to a lack of knowledge on how to solve it.

a. Misinterpretation of the keyword "5 years ago"

Figure 6

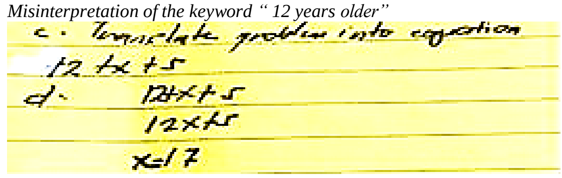
Misinterpretation of the Keyword "5 years ago"

Approximately 34% of the participants misinterpreted the keyword "5 years ago". This meant they struggled to understand that the phrase required subtracting 5, not from their current age, but from the unknown age the problem is trying to determine. Young learners, especially those new to algebra, are still transitioning from concrete to abstract thinking. While "5 years ago" is a concrete concept they can likely visualize, in algebra we represent it with variables and equations, which are more abstract [29]. Bridging this gap is where the misinterpretation often occurs.

b. Misinterpretation of the keyword "12 years older"

Figure 7

Misinterpretation of the keyword "12 years older"



Another commonly misunderstood by approximately 27% of the participants is the phrase "12 years older" which represented a constant difference between two ages. However, students often misinterpreted it as an instruction to add 12 years to someone's age as time progresses. This misunderstanding stems from a tendency to view age as changing linearly for each individual, rather than focusing on the fixed relationship between the ages.

c. Missing variable relationship

More than 10% of the participants struggled to correctly represent the relationships of variables in a problem. They often struggled to understand the concept of time shifting involved in age problems. These problems often involved past or future scenarios. Moreover, students had difficulty representing how ages changed over time while maintaining the correct relationships of variables. Students may not have fully grasped how to use variables effectively to represent unknown quantities.

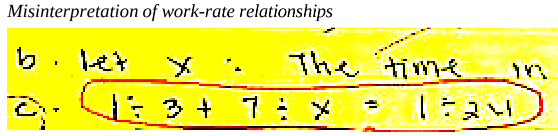
Work Problem

The work problem had also accumulated various errors among the participants. It was observed that participants exhibited multiple errors, posing significant challenges to the students.

a. Misinterpretation of work-rate relationships

Figure 8

Misinterpretation of work-rate relationships



Work problems were quite abstract in nature. Representing the concept of rate of work in this problem is often expressed as a fraction of the task done per unit of time. This level of abstraction can be difficult for students to grasp, making it harder to establish the correct relationships [30]. Moreover, students tended to have difficulty in visualizing combined work especially that this problem often involved multiple individuals or entities working together, which can be tricky to visualize. Students struggled to understand how individual work rates combine to determine the overall time taken to complete a task. Based on the results, an estimated 37% of the participants were observed to struggle in determining the correct work-rate relationship in a given problem.

b. Incorrect combined work rate

34% of the participants have been observed to struggle with combining work rates. Students often struggled with work rate problems in algebra, where they needed to combine different rates to find the total time required to complete a task. This difficulty stemmed from their inability to accurately translate the problem statement into a mathematical representation and their lack of understanding of the underlying concepts [30]. The literature suggested that students struggled to comprehend the verbal problem statement and convert it into an appropriate mathematical model [31]. They may have had trouble understanding work rate problems presented in unfamiliar contexts, as they tended to rely on memorized procedures without fully grasping the concepts [32].

c. isinterpretation of individual work rate

According to the data, 12% of students had difficulty understanding individual work rates in algebra word problems. One potential explanation is that these students may have focused on memorizing formulas and procedures without fully comprehending the underlying concepts, resulting in a lack of conceptual understanding that hindered their ability to adapt to novel problem variations [33]. This lack of understanding led them to include unexplained variables and numbers in their equations, resulting in illogical solutions. Secondly, work rate problems frequently require understanding ratios and proportions. Students might find it challenging to connect the time taken to complete a task with the concept of a rate, which involves comprehending fractions and their relationship to the overall work [34].

d. Missing variable relationship

Another pernicious error in almost all kind of word problems was the failure to represent the unknown variables. Many students formulated equations without using variables to represent the unknown quantities. Furthermore, they frequently struggled to comprehend the relationships between the different variables. This type of error was observed in approximately 15% of the responses in this study.

Mixture Problem

Mixture problems are among the most challenging word problems, with nearly 15% of students having no answer for this problem. This may be due to students feeling intimidated by the conceptual complexity of these problems or being uncertain about how to approach them. Furthermore, this issue could be linked to students' insufficient conceptual understanding, potentially resulting in a range of problem-solving difficulties [35].

a. Incorrect and incomplete representation of quantity of bills This error was the most common for this problem, with 46% of participants, nearly half the participants, making it. While most participants were able to partially account for the quantity of one type of peso bill, they also entirely overlooked including the quantity of the other type of peso bill, leading to a very inaccurate equation.

b. Missing variable relationship

This kind of error had consistently been present across all word problems. However, for this particular problem, only 5% of the participants committed this type of error. This implied that most students were able to use variables to represent unknowns in the problem and could partially determine the relationships of the unknowns, though not perfectly. For example, the problem states: $100x + 50x = 1200$. In this case, the number of peso-bills was represented correctly by the quantity "x". However, the 100-peso bill was not represented correctly by the quantity "15-x".

4.2.4 Processing Error

Processing errors refer to the mistakes committed by students while executing the calculations or procedures needed to solve a problem. This stage had also accumulated a significant number of errors made by students across different types of word problems. Participants at this stage frequently encountered significant difficulties, exhibiting escalated conceptual errors. A study found that students' inconsistent use and understanding of mathematical symbols and their definitions can result in errors when formulating algebraic equations, which then leads to further errors in solving those equations. Furthermore, errors committed in the preceding stage, such as the decoding stage, had a strong domino effect on the subsequent transforming and processing stages of Newman's error analysis framework. The domino effect of this stage began with the participants' struggle or difficulty in properly defining the key variables involved, which then led to their inability to accurately formulate the necessary mathematical equations required to solve the problem successfully. Moreover, the participants failed to correctly solve the equations they had chosen and provided during the transforming stage. This signifies a compounding effect where early mistakes hinder progress in later stages. The decoding error has been a major bottleneck for students as they progress with the solving process. This represents a critical challenge where many students struggle to transition from identifying the unknowns or variables, selecting the appropriate equation, and then completing the solving process to arrive at the correct solution. If students find it challenging to define variables, they are likely to commit errors in transforming and processing, which involves translating sentences into symbols and solving equations. This indicates significant challenges in both setting up the correct mathematical representation and executing the necessary procedures. The errors encountered in this stage can be summarized into three main categories: no answers, ambiguous or faulty solutions, and incomplete answers.

a. No Answers

“No answer” or failing to provide a solution in the solving process be considered an error, and a revealing one that. This is particularly true when the absence of an answer stems not from a lack of effort but from specific conceptual or procedural roadblocks. This implies that participants fail to grasp the problem's real-world constraints or underlying assumptions of the problem.

b. Ambiguous or faulty equation or formula

It has been observed in problems like numeracy, age, mixture, consecutive and work problems, participants have formulated different kinds of ambiguous or faulty equations which revealed serious misconceptions on the variables and their relationships, incorrect operations, failing to account the relevant conditions of the problem. Faulty equations most likely can lead to an illogical solution that involved using variables inconsistently or setting up an equation without using any variables, presenting a mathematically illogical solution. This error is an indicator that a student lacked deeper conceptual knowledge which prevented them from formulating a clear and well-reasoned solution. Addressing these gaps in conceptual knowledge is crucial to helping students develop the ability to apply mathematical principles effectively in practical situations.

c. Incomplete answer

This kind of error had been observed in problems like age and mixture problems that required one or more final answers. The error involves solving for only one variable when the problem requires finding multiple values, neglecting to translate the mathematical solution back into the context of the word problem, and failing to check if the answer makes sense within the problem's real-world constraints. While participants were able to provide a partially correct answer, the solution fails to address all aspects of the problem or provide a complete and meaningful interpretation of the results.

4.2.5 Processing Error

In Newman's Error Analysis, an encoding error occurs when a student successfully solves the mathematical part of a problem but neglects to verify their final answers for any kind of errors, such as failing to write the appropriate units, forgetting to label the final answer to describe what it represents, or missing a check of the answer against the original equation. It is quite alarming that 100% of the participants neglected this last and final stage of problem-solving. Their thinking was likely stuck in the mentality that as long as they were able to get the correct answer, they needn't bother to verify if they missed anything, like writing the appropriate units and labeling their answers or checking their answers against the original equation. This suggests a need for more explicit instructions from teachers to remind their students to complete this crucial step.

5. CONCLUSION

Based on the results of the data obtained by the researcher, it can be concluded that:

a.Participants have been observed to exhibit an escalating pattern of errors and experiencing major bottleneck during the critical stages of problem-solving. These key stages include defining variables, translating verbal descriptions into mathematical symbols, and solving the actual problem. Further analysis revealed that most participants made more mistakes starting from the decoding error stage and continuing through the processing error stage [36]. This suggests that students face growing difficulties as they progress through the problem-solving process, with errors compounding at each successive step.

a. The participants have encountered difficulties across various problem-solving stages. A significant factor was their struggle to accurately translate mathematical language with specific keywords into symbolic representations. Misinterpreting these keywords have led to cascading errors, hindering students' ability to construct accurate equations and arrive at correct solutions. This aligns with Newman's Error Analysis framework, which emphasizes the importance of accurate decoding as a foundation for successful problem-solving. When students misinterpret keywords during decoding, they start with flawed information, making subsequent steps likely incorrect. Furthermore, research has shown that students often have trouble comprehending the underlying mathematical structure embedded within word problems, which can result in the selection of inappropriate solution strategies [37].

b. Among classic word problems, participants clearly struggled most with work problems involving fractions and percentages. Existing research indicates students commonly encounter difficulties with math problems related to these concepts [38]. This was followed by problems involving age, mixture, and consecutive integers. Such problem types often necessitate the ability to translate complex relationships and constraints into algebraic expressions - a skill that can be especially demanding for learners still solidifying their algebraic reasoning abilities. Prior studies have shown these types of word problems, which require understanding the underlying structure and converting it to a mathematical representation, pose significant challenges for many students.

d. Lastly, it has been observed that most of the participant’s errors in word problems were conceptual. The study revealed that conceptual errors significantly outnumbered procedural errors among freshmen BSEd-Mathematics students. Specifically, participants struggled with understanding the problem, defining variables, and translating verbal descriptions into mathematical expressions. This suggests that many students lack a solid foundational understanding of algebraic concepts, which is crucial for their success not only in mathematics but also in their future roles as educators. If these students lacked mastery in these classic word problems, they would likely struggle in their future roles as educators. Strengthening their foundational understanding of algebraic concepts is crucial not only for their success in mathematics but also in effectively teaching such problem-solving skills to their own students.

6. RECOMMENDATIONS

Given the findings this study here are some insightful and relevant recommendations:

a. Develop targeted interventions to address the conceptual gaps that students commonly experience within Newman's Error Analysis framework. These interventions should prioritize cultivating a deeper comprehension of the underlying mathematical principles and concepts, rather than solely focusing on procedures or rote memorization. Strengthen the instruction on teaching students how to effectively translate essential and critical keywords that frequently cause confusion and errors. Move beyond rote memorization of procedures and instead focus on building a robust understanding of algebraic concepts, such as variables, expressions, equations, and the relationships between them. Employ a combination of metacognitive strategies, including self-monitoring, self-explanation, and error analysis, to empower students to take a more active role in their own learning process.

b. Engage students in diverse word problems to broaden their conceptual knowledge and develop a deeper understanding of the underlying mathematical concepts. By doing so, they will be able to see connections between different approaches and develop more efficient strategies. These strategies will help them gain a deeper understanding of problem-solving and become more adept at tackling a range of mathematical challenges. Integrate opportunities for students to engage in error analysis, where they reflect on their own mistakes, identify the underlying causes, and then work on correcting their understanding.

c. Empower teachers through professional development by deepening their own conceptual knowledge and develop pedagogical approaches specifically for effective teaching word problem-solving. Provide teachers with strategies to identify and diagnose common student errors, and then guide them in designing targeted interventions to address those errors [39].

d. Develop a comprehensive set of resources for teachers to support effective word problem-solving instruction and assessment. This should include detailed lesson plans that guide teachers through the process of teaching problem-solving strategies, a diverse collection of sample word problems to use in the classroom, assessment rubrics to evaluate students' problem-solving abilities, and video demonstrations showcasing exemplary teaching practices in action. These resources should provide teachers with practical tools and guidance to enhance their students' proficiency in solving complex word problems across various mathematical domains.

e. Providing targeted support to these freshmen BSED-Math students and focus in areas of problem solving where students struggled the most. For example is the predominance of conceptual errors highlights the necessity for targeted instructional strategies that focus on enhancing conceptual understanding. As future mathematics educators, these students must be equipped with a robust grasp of algebraic principles to effectively teach their students. Research indicates that a strong foundation in basic algebra concepts significantly impacts students' ability to tackle more advanced mathematical topics. Therefore, addressing these conceptual gaps by providing targeted instructions early on can prevent a cycle of misunderstanding that may persist throughout their academic careers.

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